Applied Energy 88 (2011) 5193–5199

Contents lists available at ScienceDirect

Applied Energy

journal homepage: www.elsevier.com/locate/apenergy

Analytical model of parallel thermoelectric generator

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ARTICLE INFO

Article history:
Received 12 April 2011
Received in revised form 20 July 2011
Accepted 24 July 2011

Keywords:
Parallel TEG
TE module
Analytical model
Output performance
Contact effect

ABSTRACT

The paper studied the performances of parallel thermoelectric generator (TEG) by theoretical analysis and experimental test. An analytical model of parallel TEG was developed by theoretical analysis and calculation, based on thermodynamics theory, semiconductor thermoelectric theory and law of conservation of energy. Approximate expressions of output power and current of parallel TEG were deduced by the analytical model. An experimental system was built to verify the model. The results indicate that only when all of the thermoelectric modules (TE modules) in the parallel TEG have the same inherent parameters and working conditions, the parallel properties of the TEG are the same as that of common DC power. The existence of contact resistance is just like the increase of the TE module's internal resistance, which leads to the deceases of output power. The thermal contact resistance reduces the output power by reducing the temperature difference between the two sides of the thermocouples. The results derived from the model are basically consistent with the experimental results, the model is suitable for the performance researching and designing of parallel TEG.

1. Introduction

A TEG produces a voltage when there is a temperature difference between the hot-side and cold-side of TEG by thermoelectric effect, which means that the thermal energy is converted directly into electric energy by thermoelectric effect. Thermoelectric effect includes Seebeck effect, Peltier effect and Thomson effect, it also accompanies with other effects, such as Joulean effect and Fourier effect.

Thermoelectric generation is a technology for directly converting thermal energy into electrical energy, it has no moving parts, is compact, quiet, highly reliable and environmentally friendly. Because of these merits, it is presently becoming a noticeable research direction [1].

In the aspect of system analysis and optimization, Rowe and Gao [2] provided a practical procedure for optimizing TE module geometry guided by the "economic factor". Chen et al. [3] explored the performance of the TEG with multi-element thermoelectric equipment by assuming that the heat transfer between the thermoelectric generator and the external heat reservoirs obeys the linear phenomenological law used in the irreversible thermodynamics. Palacios et al. [4] showed a methodology to extract thermoelectric internal parameters from the information provided by performance curves, allowing scientists to predict the performance of the module at any working condition. Chen et al. [5] built a model of a two-stage semiconductor thermoelectric-generator with external heat-transfer. Yamashita [6,7] built thermal rate equations by taking the linear and non-linear components in the temperature dependences of the Seebeck coefficient, the electrical resistivity and thermal conductivity. Gao and Rowe [8] fabricated a novel tube-shape TE module from four ring-shaped thermo elements, and established an analytical model of the tube-shape TE module, the result indicated that a tube-shape TE module could achieve similar performance to that of a conventional plate-like module, and had an advantage in applications when heat flowed in a radial direction.

In the aspect of thermoelectric technology application in industry, Hsu et al. [9], Masahide et al. [10], Yodovard [11], Yu and Chau [12], and Xu et al. [13] applied the thermoelectric generation technology to the environment-friendly vehicle, both waste heats from vehicle exhaust and engine were recovered into electrical power to provide auxiliary power for the vehicle. Chen et al. [14] presented a system efficiency analysis and impact evaluation of the application of thermoelectric power cycle to thermal energy systems, especially Combined Heat and Power production. In order to investigate viability and further performance of the TEG for waste heat recovery in industry area, Gou et al. [1] and Niu et al. [15] constructed a low-temperature waste heat thermoelectric generator setup, the testing results and discussion showed the promising potential of using TEG for low-temperature waste heat recovery, especially in industrial fields.

In the aspect of new thermoelectric materials research and development, nowadays, a large number of works concerning ther-

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doi:10.1016/j.apenergy.2011.07.041

moelectricity focus on how to improve heat-to-electricity conversion efficiency of thermoelectric materials, that is, how to improve the thermoelectric figure of merit *ZT* of these materials. Nanotechnology [16,17], novel technology in solid state physics and semiconductor physics [18,19] are employed to explore this exciting field.

Despite these promising results, wide application of TEG has yet been limited because of its relatively low heat-to-electricity conversion efficiency. However, compared with conventional methods, thermoelectric generation technology has great advantage in some individual areas. For example, in the area of low-grade waste heat recovery, as it cannot be recycled effectively by conventional methods, TEG appears to have advantages due to its entire solid-state energy conversion mode. Furthermore, considering waste heats are low-cost and even no-cost resources, the low heat-to-electricity conversion efficiency is no longer a major problem, the focus of our concern is how to get larger output power of a TEG.

As the output power of the TEG composed of only one TE module is very small, to obtain higher output power, it is feasible to make a TEG by multiple TE modules in series connection or parallel connection (for short, named series TEG and parallel TEG in this paper, respectively). Zhang et al. [20] studied the performances of series–parallel connection for TE modules by experiment and numerical calculation, the results showed that when all the modules had the same parameters, and worked in the same condition, the series–parallel properties of the TEG were the same as the series–parallel properties of common DC power. However, to a multiple modules TEG, it is hard to guarantee every TE module works in the same conditions. For example when the thermoelectric technology is used in the waste heat recovery of industry furnaces, due to the different surface temperature of every part of furnaces, it will cause the different hot-side temperature of every TE module. Therefore, there are more components in the multiple modules TEG, and its performance analysis is more complex than the one module TEG. As previous researches about multiple modules TEG focused on the performance analysis of the TEG that all TE modules are in the same working conditions, when each TE module of multiple modules TEG is in different working conditions, it is necessary to build an analytical model of series TEG and an analytical model of parallel TEG.

The output performance of series TEG has been studied by us recently [21]. In this research, the output performance of parallel TEG is studied, an analytical model of parallel TEG is proposed based on thermodynamic theory, semiconductor thermoelectric theory, and law of conservation of energy. Approximate expressions of output power and current of parallel TEG are deduced by the analytical model. Compared with the previous researches, the model takes into account much more influencing factors, such as thermal contact resistance, contact resistance, and temperature of heat source and heat sink of every TE module. Just as the previous researches, the model can be used to analyze the output performance of parallel TEG when all TE modules work in the same conditions. But the model can also be used to analyze the output performance of parallel TEG when all TE modules work in different conditions, which cannot be analyzed by previous researches. The model can be used for design and analysis of parallel TEG accurately.

The purpose of this paper is to build up an analytical model of parallel TEG by theoretical analysis, to investigate how contact effect reduces the output power of parallel TEG, and further to verify the model by experimental test.

2. Model description

2.1. Model structure

The structure of a typical one TE module TEG is shown schematically in Fig. 1, it consists of a TE module, a heat source and a heat sink. A TE module consists of over 100 P-N thermocouples, conductive tabs, and two ceramic substrates. The thermocouples are configured so that they are connected electrically in series, but thermally in parallel. Ceramic substrates provide the platform for the thermocouples and the small conductive tabs that connect them. Heat source, thermocouples, conductive tabs, ceramic substrates, and heat sink thus form a layered configuration.

When the temperature of the heat source and the heat sink cause the temperature difference between the hot-side and cold-side of thermocouples, five thermoelectric effects will occur. Therefore, a current *I* will flow through an external load resistance r_L because of the temperature difference.

The structure of a parallel TEG is shown schematically in Fig. 2. Compared to the one TE module TEG, it consists of multiple TE modules, heat sources and heat sinks, and only one load resistance.

The parallel TEG consists of *n* TE modules which are consecutively numbered from 1 to *n*, T_{ih} and T_{ic} are the heat source temperature and heat sink temperature of the *i*th ($1 \leqslant i \leqslant n$) TE module, respectively, T_{i1} and T_{i2} are the thermocouple's hot-side temperature and cold-side temperature of the *i*th TE module, respectively.

2.2. Mathematical modeling

The following assumptions are made to simplify the complex problem of modeling:

(a) All the TE modules of the parallel TEG are the same model, which means that they have the same inherent parameters.
(b) The heat conduction between thermocouples is ignored, as the heat conduction from hot-side to cold-side within the thermocouples as axial heat conduction will be dominant.
(c) The Seebeck coefficient, thermal conductivity and resistivity of semiconductor thermocouples are constants.
(d) On the assumption that Seebeck coefficient remains unchanged, the influence of Thomson effect is not considered.

There are *n* TE modules in the parallel TEG shown in Fig. 2. To a TE module, let *m* be the number of thermocouples, α (VK^{-1}) be the Seebeck coefficient of a thermocouple, *K* (WK^{-1}) be the thermal conductance of all thermocouples, *r* (Ω) be the internal resistance of a TE module, r_e (Ω) be the contact resistance of a TE module which including the contact resistance between thermocouples and conductive tabs, the internal resistance of conductive tabs. As *m*, α, *K*, *r* and r_e are all inherent parameters of TE modules, according to the previous assumption, they have the same value to each TE module.

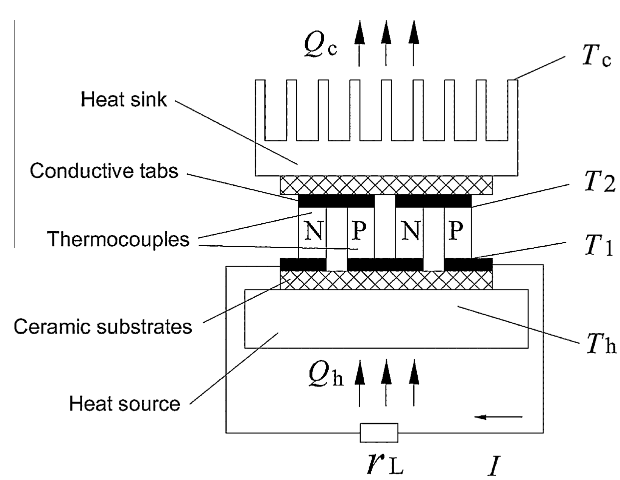

Fig. 1. Schematic diagram of a typical TEG.

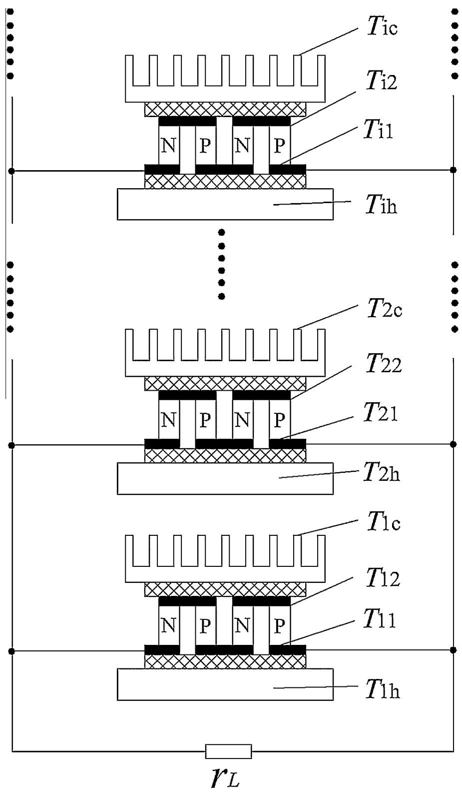

Fig. 2. Schematic diagram of a parallel TEG.

The diagram of thermal conductance and thermal resistance of every part of a TE module is shown schematically in Fig. 3.

In Fig. 3, K_{ih} (WK^{-1}) and K_{ic} (WK^{-1}) are the total hot-side and cold-side thermal conductance of the ith TE module, respectively. K_1 (WK^{-1}) is the sum of thermal conductance of hot side or cold side conduct taps, K_2 (WK^{-1}) is the thermal conductance of hot side or cold side ceramic substrate. R_1 (KW^{-1}) is the sum of the thermal resistance between the thermocouples and conduct taps on either side. R_2 (KW^{-1}) is the sum of the thermal resistance between the conduct taps and ceramic substrate on either side. R_{ih} (KW^{-1}) is the thermal contact resistance between the heat source and hot-side ceramic substrate, it reflects the contact condition between the heat source and the TE module. R_{ic} (KW^{-1}) is the thermal contact resistance between the heat sink and cold-side ceramic substrate, it reflects the contact condition between the heat sink and the TE module. R_{i3} (KW^{-1}) is the thermal resistance of the heat sink, it is related to the cooling method adopted, the heat sink structure, application environment, and so on.

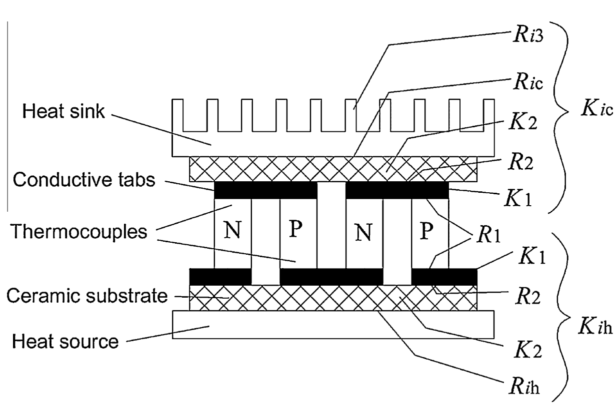

Fig. 3. Schematic diagram of thermal conductance and thermal resistance of every part of a TE module.

Since K_1, K_2, R_1 and R_2 are the inherent parameters of TE module, they have the same value to every TE module. However, R_{ih} and R_{ic} are not the inherent parameters of TE module, they reflect the contact condition between the TE module and the heat source or heat sink, respectively, and it cannot be guaranteed that all contact surfaces have the same contact condition for different TE modules. For example, when TEG is used to convert the surface heat of boiler into electric energy, the boiler surface will be the heat source of TEG, as the boiler surface is not smooth, the contact condition between boiler surface and different TE module is not the same. So it is possible that there are different R_{ih} and R_{ic} in different TE modules. Besides, as R_{i3} is related to the application environment, it is also possible that there are different R_{i3} in different TE modules.

It can be seen form Fig. 3 that K_{ih} and K_{ic} consist of many parts of thermal conductance and thermal resistance, so K_{ih} and K_{ic} can be expressed as

$$\frac{1}{K_{ih}} = \frac{1}{K_1} + \frac{1}{K_2} + R_1 + R_2 + R_{ih}, \tag{1}$$

$$\frac{1}{K_{ic}} = \frac{1}{K_1} + \frac{1}{K_2} + R_1 + R_2 + R_{i3} + R_{ic}. \tag{2}$$

For commercial TE module, the value of K_1, R_1 and R_2 is generally very small, the influencing factors of thermal resistance of hot-side and cold-side are mainly K_2, R_{i3}, R_{ih} and R_{ic}.

It is assumed that every TE module in the parallel TEG works on different conditions, such as different heat source temperature, and different heat sink temperature. Let Q_{ih} (W) present the heat that absorbed from its heat source by the ith TE module per unit time, and Q_{ic} (W) present the heat that released to its heat sink per unit time.

According to Fourier's law, Q_{ih} and Q_{ic} can be expressed as

$$Q_{ih} = K_{ih}(T_{ih} - T_{i1}), \tag{3}$$

$$Q_{ic} = K_{ic}(T_{i2} - T_{ic}). \tag{4}$$

In addition, Q_{ih} consists of three parts of heat, Pletier heat on the hot-side of thermocouples, Joule heat generated by internal resistance of the TE module as the current flow through the TEG circuit, and conduction heat from the hot-side to the cold-side of the thermocouples [22,23]. So Q_{ih} can also be expressed as

$$Q_{ih} = m\alpha T_{i1} I_i - \frac{1}{2} I_i^2 (r + r_e) + K(T_{i1} - T_{i2}), \tag{5}$$

where I_i is the current flowing in the ith TE module.

Similarly, Q_{ic} can also be expressed as

$$Q_{ic} = m\alpha T_{i2} I_i + \frac{1}{2} I_i^2 (r + r_e) + K(T_{i1} - T_{i2}). \tag{6}$$

Let the current flowing through the load be I, according to Kirchhoff's current law, in a parallel circuit the total current is equal to the sum of the currents in each individual branch, the current I can be expressed as

$$I = I_1 + I_2 + \cdots I_i + \cdots + I_n = \sum_{i=1}^{n} I_i. \tag{7}$$

Let the load resistance of the parallel TEG be r_L (Ω), the output power of the parallel TEG be P (W), and thus P can be expressed as

$$P = I^2 r_L. \tag{8}$$

In addition, according to the law of energy conservation, P can also be expressed as

$$P = \sum_{i=1}^{n}(Q_{ih} - Q_{ic}). \tag{9}$$

Combining Eqs. (5)–(9), we get

$$\left(\sum_{i=1}^{n} I_i\right)^2 r_L = \sum_{i=1}^{n}\left[m\alpha(T_{i1} - T_{i2})I_i - I_i^2 r\right]. \tag{10}$$

Combining Eqs. (3), (4), and (10), we will get a cubic equation group of T_{i1} and T_{i2}, it is so complicated that no analytical solution can be found.

Since every TE module has the same internal resistance r, in order to get the approximation solution of parallel TEG, the following approximate derivation can be done (see Appendix A). The parallel TEG which composed of n TE modules can be seen as n independent "small TEGs", each "small TEG" works on its respective heat source and heat sink, the load resistance of each "small TEG" is the n times as large as that of the parallel TEG, that is nr_L, the sum of currents in "small TEGs" is equal to the current of the parallel TEG. Therefore, to the ith TEG, the temperature difference between the hot-side and the cold-side of the thermocouples ΔT_i and the current I_i are [24]

$$\Delta T_i = T_{i1} - T_{i2} = \frac{1}{\frac{m^2\alpha^2 B_i}{r+r_e+nr_L} + A_i}(T_{ih} - T_{ic}), \tag{11}$$

$$I_i = \frac{m\alpha\Delta T_i}{r + r_e + nr} = \frac{m\alpha(T_{ih} - T_{ic})}{m^2\alpha^2 B_i + A_i(r + r_e + nr_L)}, \tag{12}$$

where

$$A_i = 1 + \frac{K}{K_{ih}} + \frac{K}{K_{ic}}, \tag{13}$$

$$B_i = \frac{T_{ih}}{K_{ic}} + \frac{T_{ic}}{K_{ih}}. \tag{14}$$

Therefore, the total current of the parallel TEG is

$$I = \sum_{i=1}^{n} \frac{m\alpha(T_{ih} - T_{ic})}{m^2\alpha^2 B_i + A_i(r + r_e + nr_L)}. \tag{15}$$

The output power of the parallel TEG is

$$P = \left[\sum_{i=1}^{n} \frac{m\alpha(T_{ih} - T_{ic})}{m^2\alpha^2 B_i + A_i(r + r_e + nr_L)}\right]^2 r_L. \tag{16}$$

When the model and parameters of TE modules are selected, the heat source temperature and the heat sink temperature of each TE module are known, and the thermal contact resistances are measured, let $\frac{\partial P}{\partial r_L} = 0$, the maximum output power and match load of the parallel TEG can be got. However, as the Eq. (16) is very complicated, it is impossible to get the analytical expression of maximum output and match load. In practical application, when the values of all parameters are determined, the maximum output power and match load of parallel TEG can then be calculated by above method.

3. Discussion

3.1. Output performances under the ideal condition

In some cases, it is possible that all TE modules of parallel TEG have the same working conditions and inherent parameters. The radioisotope TEG, which is used in the fields of military and space exploration serves as a good example, it can be guaranteed that each TE module has the same heat source and heat sink by design, and it can also be guaranteed that each TE module has good contact with the heat source and heat sink by production process control. Consequently, they have the same thermal contact resistance R_{ih} and R_{ic}, according to Eqs. (1) and (2), it can be thought that all TEG modules have the same thermal conductance on hot-side and cold-side. Therefore, under this ideal condition, as all TE modules of parallel TEG have the same working conditions and inherent parameters, the process and result of performance analysis can be simplified.

Let T_h be the heat source temperature, T_c be the heat sink temperature, K_h be the total hot-side thermal conductance of a TE module, K_c be the total cold-side thermal conductance of a TE module, and thus, Eq. (16) can be simplified as

$$P = \left[\frac{nm\alpha(T_h - T_c)}{m^2\alpha^2 B + A(r + r_e + nr_L)}\right]^2 r_L, \tag{17}$$

where

$$A = 1 + \frac{K}{K_h} + \frac{K}{K_c}, \tag{18}$$

$$B = \frac{T_h}{K_c} + \frac{T_c}{K_h}. \tag{19}$$

Let $\frac{\partial P}{\partial r_L} = 0$, the maximum output power P_{max} and match load r_m of the parallel TEG are got as

$$P_{max} = \frac{nm^2\alpha^2(T_h - T_c)^2}{4m^2\alpha^2 AB + 4(r + r_e)A^2}, \tag{20}$$

$$r_m = \frac{r + r_e}{n} + \frac{m^2\alpha^2 B}{nA}. \tag{21}$$

Similarly, Eq. (15) can be simplified as

$$I = \frac{\frac{m\alpha}{A}(T_h - T_c)}{\frac{m^2\alpha^2 B + A(r+r_e)}{nA} + r_L}. \tag{22}$$

In Eq. (22), r_L is the load resistance of the parallel TEG, $\frac{m\alpha}{A}(T_h - T_c)$ can be seen as the open-circuit voltage of the parallel TEG, $\frac{m^2\alpha^2 B + A(r+r_e)}{nA}$ can be seen as the internal resistance of the parallel TEG.

However, to the one module TEG, its current is [24]

$$I = \frac{\frac{m\alpha}{A}(T_h - T_c)}{\frac{m^2\alpha^2 B + A(r+r_e)}{A} + r_L}. \tag{23}$$

Thus, to the one module TEG, $\frac{m\alpha}{A}(T_h - T_c)$ is the open-circuit voltage, $\frac{m^2\alpha^2 B + A(r+r_e)}{A}$ is the internal resistance, r_L is the load resistance.

Compare the Eq. (22) with (23), it can be seen that the open -circuit voltage of the parallel TEG is as large as that of the one module TEG, and the internal resistance of parallel TEG is n times as small as that of the one module TEG. That is to say, when all the modules have the same parameters, and work in the same condition, the parallel properties of the parallel TEG are the same as the parallel properties of common DC power, it conforms to the results of Zhang et al. [20].

3.2. Influence of contact effect on output performance

The contact resistance r_e is the inherent parameter of TE module. The thermal contact resistance has two types of parameter. One is the inherent parameters of TE module, K_1, K_2, R_1, and R_2. The other is the structural parameters of parallel TEG which inflect the contact conditions between the TE module and the heat source or heat sink, namely R_{ih} and R_{ic}.

From Eqs. (15) and (16), the existence of contact resistance r_e is just like the increase of the TE module's internal resistance, which

may lead to the deceases of current I and output power P. Therefore, the effect of r_e on output power cannot be neglected, though its value is generally very small to commercial TE module.

The increase of thermal contact resistance means the decreases of K_{ih} and K_{ic}, according to Eqs. (13) and (14), A_i and B_i will increase with the increase of thermal contact resistance. According to Eq. (11), the increases of A_i and B_i will lead to the decrease of ΔT_i, it means the decrease of temperature difference between the two sides of the thermocouples. Consequently, according to Eqs. (15) and (16), the current I and output power P will decrease. In a word, the thermal contact resistance reduces the output power by decreasing the temperature difference between the two sides of the thermocouples.

Overall, the existence of the contact resistance and thermal contact resistance will lead to the decrease of parallel TEG's output power. The result is the same as that of the one TE module TEG [24] and series TEG [21].

In extreme cases, when the contact resistance and thermal resistance of hot-side and cold-side are so small that they can be neglected, thus, r_e is zero, and K_{ih} and K_{ic} tend to infinity, so Eq. (16) can be simplified as

$$P = \left[\sum_{i=1}^{n} \frac{m\alpha(T_{ih} - T_{ic})}{r + nr_L}\right]^2 r_L. \tag{24}$$

Let $\frac{\partial P}{\partial r_L} = 0$, the maximum output power P_{max} and the match load r_m are got as

$$P_{max} = \frac{m^2 a^2}{4nr}\left[\sum_{i=1}^{n}(T_{ih} - T_{ic})\right]^2, \tag{25}$$

$$r_m = \frac{r}{n}. \tag{26}$$

A simple parallel TEG composed of two TE modules is used to analyze the influence of contact effect on output power. The TE modules are the TEP1 power module of Xiamen Taihuaxing Trading Co. Ltd., its parameters are

$$m = 126, \quad \alpha = 0.000375\ \mathrm{V\,K^{-1}}, \quad K = 0.476\ \mathrm{W\,K^{-1}}, \quad r = 3.0\ \Omega$$

It is assumed that the two TE modules have the same contact resistance and thermal contact resistance, set the contact resistance r_e be 0.1 Ω, the thermal conductance of hot-side and cold-side be all 10 $\mathrm{WK^{-1}}$. To one of the TE module, T_{1h} is 400 K, T_{1c} is 300 K, to another module, T_{2h} is 450 K, T_{2c} is 320 K.

In range of 0–3 Ω, adjust the load resistance of parallel TEG, and the current and output power are calculated under two conditions of neglecting the contact effect and considering the contact effect. The result is displayed in Fig. 4.

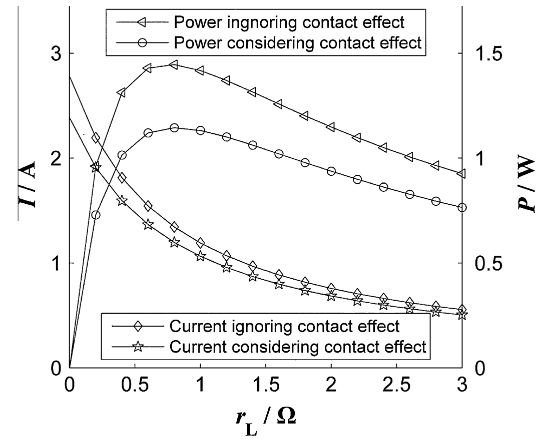

Fig. 4. Current I and output power P as a function of load r_L.

It is easy to see from Fig. 4 that the existence of contact effect will reduce the current and output power, therefore, in order to improve the output power, the influence of contact effect must be reduced as far as possible.

In the design and production of TE module, using high conductivity materials (usually copper) as the material of conductive tabs is a feasible method for reducing properly the resistance of conductive tabs, it can reduce the contact resistance of TE module. In order to reduce the thermal contact resistance, a common method is to use high-conductivity ceramic materials (usually alumina ceramic) as the substrate of TE module, in the condition that the ceramic substrate is strong enough, reducing properly its thickness will make the thermal contact resistance decrease. Besides, according to the different working conditions, using suitable cooling method and heat sink structure is very necessary to reduce the thermal resistance of heat sink.

In the practical application, improving the surface finish and flatness of heat source and heat sink, increasing properly the pressure between the TE module and the heat source or heat sink can increase the real contact area. Besides, high-conductivity thermal interface material (e.g., heat conductive silicone grease, thermally conductive pad, phase-change material) is often used to ensure good contact between surfaces. These methods will cause the decrease of R_{ih}, R_{ic} in Eqs. (1) and (2), thus, the thermal contact resistance will decrease.

4. Experimental results

In order to verify the parallel model established in this research, a parallel TEG experimental system is built, the schematic of the parallel TEG performance testing system is shown in Fig. 5.

The parallel TEG consists of two TE modules which are numbered as TE-1 and TE-2, respectively. The TE module are the TEP1 power module of Xiamen Taihuaxing Trading Co. Ltd., its inherent parameters are given in Section 3.2 of this paper. The heat source of each TE module is a heating plate made of aluminum, its temperature can be adjusted by the voltage on the electric heating tube. The heat sink of each TE module is a water-cooling plate made of

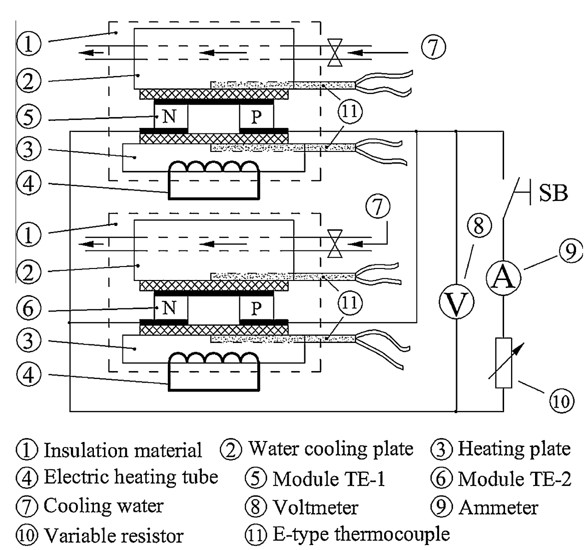

Fig. 5. Schematic of the parallel TEG performance testing system.

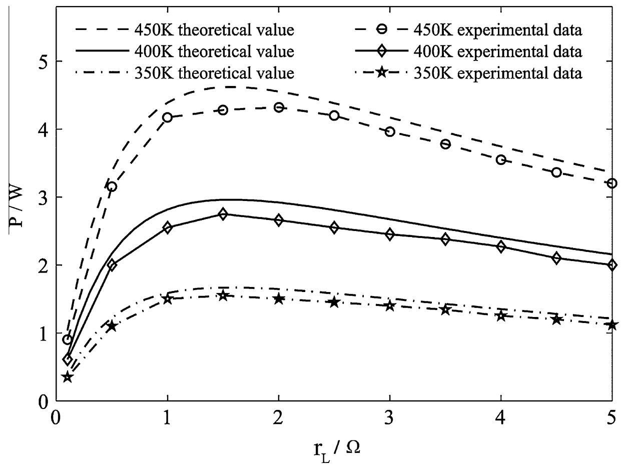

Fig. 6. Output power P as a function of load r_L under different heat source temperature.

copper, its temperature can be adjusted by the flow and temperature of cooling water.

Some U-shaped slots are cut on the heating plates surface and water-cooling plates surface which contact with the TE modules, the dimension of slots is 1.0 mm × 1.0 mm. 1.0 mm-diameter E-type thermocouples (nickel–chromium/copper–nickel) are inserted into these slots to measure the temperatures, the accuracy of measurement is ±0.1 K. To reduce the thermal contact resistance, the interfaces of the heat source and the TE module, the TE module and heat sink are all coated with heat conductive silicone grease, it can be thought that all the interfaces have good contact addition. At the same time, it is assumed that thermal resistance of heat sink is so small that it can be neglected, so the thermal conductance K_h and K_c are all 10 WK^{-1} which mainly consider the thermal resistance of ceramic substrate. To minimize the bad effects of convection and radiation, the TE module, the heat source and heat sink are all wrapped with insulation material together.

During the test, the heat source temperature and heat sink temperature of module TE-1 are kept to 400 K and 300 K, respectively. The heat source temperature of module TE-2 has the different value, 350 K, 400 K and 450 K, while its heat sink temperature is kept unchanged to 300 K. In the different heat source temperature of module TE-2, the value of load r_L is adjusted from 0.1 Ω to 5 Ω, at the same time, the output power of the TEG system is carefully measured. Fig. 6 displays the output power of the parallel TEG as a function of load under different heat source temperature.

As shown in Fig. 6, the results derived from the model are basically consistent with the experimental results. However, the theoretical value is slightly larger than the experimental results, the deviation between the theory and experimental results may be due to the previous assumption which ignores the heat conduction between the thermocouples. The temperature difference between the two sides of the thermocouples will decrease when the heat conduction between the thermocouples is considered, then results the decreases of output power.

5. Conclusion

A novel analytical model of parallel TEG was developed based on thermodynamic theory, semiconductor thermoelectric theory, and law of conservation of energy, the equations of output power and current of parallel TEG were derived. The model takes into account much more parameters than the one TE module TEG, such as temperature of heat source and heat sink, contact resistance, and thermal contact resistance of each TE module in parallel TEG. The results show that only all TE modules in the parallel TEG have the same inherent parameters and work in the same condition, the parallel properties of the parallel TEG are the same as the parallel properties of common DC power. Besides, contact effect will reduce the output power of parallel TEG. The existence of contact resistance is just like the increase of the TE module's internal resistance, which leads to the deceases of output power. The thermal contact resistance reduces the output power by reducing the temperature difference between the two sides of thermocouples. In the practical application, as the parameters of TE module are fixed, the main method of reducing the influence of contact effect is reducing the thermal contact resistance between the TE module and the heat source or heat sink.

The results derived from the model are basically consistent with the experimental results, it demonstrates that the analytical model of parallel TEG works well for predicting the output performance. However, the theoretical value is slight larger than the experimental results because of the assumption which ignores the heat conduction between the thermocouples.

The analytical model of parallel TEG is a useful tool for those who want to design parallel TEG, since it will give them an idea about how to predict the output performance of parallel TEG.

Acknowledgments

This work is supported by National Natural Science Foundation of China (Grant No. 50876116) and Scientific Research Fund of Hunan Provincial Education Department (Grant No. 10c1193).

Appendix A. Approximate derivation of parallel TEG circuit

The parallel TEG circuit can be seen approximately as a simple parallel circuit shown in Fig. A.1.

In the simple parallel circuit, there are n power supplies in parallel, and every power supply has the same internal resistance r. In accordance with the Kirchhoff's law and Ohm's law, we can get the following equations:

$$I = I_1 + I_2 + \cdots I_i + \cdots + I_n = \sum_{i=1}^{n} I_i \tag{A.1}$$

$$Ir_L = U_i - I_i r \tag{A.2}$$

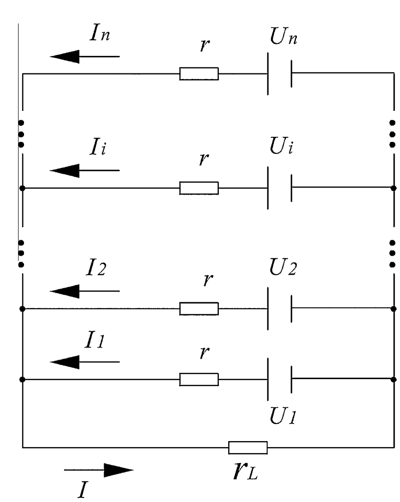

Fig. A.1. Simple parallel circuit diagram.

where I is the current flowing through the load r_L, I_i is the ith branch current, U_i is the electromotive force of the ith power supply.

Reforming Eq. (A.2) yields

$$I_i = \frac{U_i - Ir_L}{r} \tag{A.3}$$

Substituting Eq. (A.3) into Eq. (A.1) yields

$$I = \frac{\sum_{i=1}^{n} U_i - nIr_L}{r} \tag{A.4}$$

Reforming Eq. (A.4) yields

$$I = \sum_{i=1}^{n} \left(\frac{U_i}{r + nr_L} \right) \tag{A.5}$$

Combining Eq. (A.1) and Eq. (A.5) yields

$$I = \sum_{i=1}^{n} I_i = \sum_{i=1}^{n} \left(\frac{U_i}{r + nr_L} \right) \tag{A.6}$$

According to Eq. (A.1), the current I is equal to the sum of total branch current I_i, that is to say, in order to get I, the value of every I_i does not matter as long as the sum of total I_i is unchanged. According to Eq. (A.6), the sum of total $\frac{U_i}{r+nr_L}$ is equal to the current I too, therefore, a hypothesis that replacing I_i with $\frac{U_i}{r+nr_L}$ can be made, though I_i is not necessarily equal to $\frac{U_i}{r+nr_L}$, that is

$$I_i = \frac{U_i}{r + nr_L} \tag{A.7}$$

Eq. (A.7) is a formula to solve the current of a simply circuit based on Ohm's law, r is the internal resistance of the power supply, and nr_L is the load impendence of the simply circuit. Therefore, in order to get the load current I of the parallel circuit shown in Fig. A.1, the parallel circuit can be divided into n simply circuits, and the internal resistance of every simply circuit is r, the load is nr_L.

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